RELATIONSHIP FINANCE, INFORMATIONAL RENT, AND OBSERVATIONAL INDISTINGUISHABILITY FROM SHORT-TERMISM*

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Abstract

Relationship intermediaries specialise in alleviating information asymmetry, but the literature remains vague on precisely what information they acquire and how they put such information to use. This paper characterises explicitly a particular class of signal extraction and studies the optimal funding policy that maximises the associated informational rents. Because it strikes the optimal trade-off between present and future periods, this funding policy cannot be labelled short-termist. Yet if the relationship intermediary's ROC curve is kinked, this intertemporally optimal funding policy may nevertheless appear indistinguishable from short-termism to the uninformed outside observer.

Keywords: Relationship Intermediation; Informational Rent; Quasi-Option Value; Short-Termism; Signal Detection; Receiver Operating Characteristics (ROC) curve

JEL classification: G21, G24, L14

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1 Introduction

Relationship intermediaries make long-term funding possible by alleviating asymmetric information with specialised monitoring routines and by facilitating flexible adaptation to altered circumstances with renegotiation and informed discretion. Whereas it is generally understood how renegotiation and informed discretion may smooth the supply of funding through periods of near and actual financial distress, less is known about how relationship intermediaries' specialised information acquisition activities contribute directly to long-termism. Indeed the literature on relationship intermediation is also recognised to be somewhat vague on the precise type(s) of information acquired, the manner in which this information is acquired and how this information is put to use (Boot, 2000).

Long-termism and short-termism are often defined as complements, the latter referring to a distortion resulting from either an excessively short planning horizon or an excessively high discount rate on distant cash flows, the former referring to the absence of such distortion. Under this convention long-termism is looked upon as a uniform 'good thing', whereby the more weight placed on gains in the distant future the better. This convention does not explicitly admit the possibility of overweighting distant gains, and hence excludes, by categorical omission, consideration of the inefficiency attendant to such overweighting.

Terminological conventions in the public sphere aside, an optimal funding policy must strike a balance, extending funding up to the point where marginal expected next-period losses equal the marginal expected subsequent benefits thereby secured. Extending funding beyond this point places excessive weight on the distant future relative to the proximate future, constituting an inefficiency of equal concern to the placing of excessive weight on the proximate future relative to the distant future. The optimal trade-off also contrasts with the notion that long-dated funding contracts are necessary to escape short-termism. Nevertheless even if relationship funding is not explicitly staged on a quarterly or semi-annual basis, as in the case of venture capital investment, then the same effect is achieved by (i) the presence of material adverse conditions clauses in nominally long-term loan contracts, (ii) covenants and (iii) quarterly reporting requirements (Aivazian, Booth and Cleary, 2003). The multi-period structure of the model developed here captures the insight that the staging of funding contracts – whether explicitly or only implicitly – is an inherent feature of relationship funding.

This paper implements a particular refinement of the relationship intermediary's information

acquisition and signal extraction apparatus. The operation of this apparatus affords the relationship intermediary informational rents from each of its funding relationships. In the model elaborated here, the relationship intermediary's objective is to maximise the value of these informational rents, and this necessarily involves striking the optimal trade-off between proximate and distant returns. We study the effects of the form of the signal extraction apparatus on the optimal funding policy, and identify conditions under which the intertemporally optimal funding policy may appear observationally indistinguishable from a short-termist policy. If the relationship intermediary's Receiver Operating Characteristics (ROC) curve is kinked, the optimal funding policy sets a constant investment approval probability over a non-degenerate interval of project lifetime. Within this interval – and given that the internal structure and operation of the signal extraction apparatus remains unobserved by the outsider – the apparent insensitivity of the funding policy may (mistakenly) be attributed to an excessively high discount rate or an excessively short planning horizon. We formalise this result as an indistinguishability theorem.

In addition to stressing specialised monitoring skills, the existing literature views relationship intermediaries as specialised reorganisers and facilitators of financial flexibility (Boot, 2000). Because of the localised informational advantage gained through monitoring, contract renegotiation is less costly for relationship intermediaries than for arm's-length financiers or syndicates of arm's-length intermediaries (Preece and Mullineaux, 1996). Through renegotiation and re-configuration of funding terms when necessary, relationship intermediaries pro-actively contribute toward ensuring the continuation of funding relationships. In this way, relationship intermediaries' scope for flexibility is associated with uninterrupted, ex post long-term funding.

Yet in order to focus squarely on effects associated with informational rent extraction, the model developed here abstracts from renegotiation. Furthermore, the model also abstracts from collateral and security; if the firm fails in particular period, the relationship intermediary loses the entire principal committed to the firm in that period as well as the discounted expected value of informational rents from subsequent periods. Thus, in a strict sense, the modelling framework supports inferences regarding non-secured renegotiation-free financing. The present modelling framework bears some resemblance to elements of Rajan (1992) and Sharpe (1990), but its purpose and direction of development is different. Finally, to avoid handicapping the relationship intermediary's ability to supply ex post long-term funding and to avoid complicating its analytical underpinnings, the model permits the relationship intermediary to appropriate the

full value of informational rents.

The rest of this paper is organised as follows. Section 2 introduces the basic concepts and model structure that is used later on. Section 2.1 sets out a two-period model of relationship investment; section 2.2 characterises the relationship intermediary's information acquisition apparatus; section 2.3 formalises the relationship intermediary's signal extraction as an application of Signal Detection Theory; and section 2.4 formalises the relationship intermediary's gains made possible by its information acquisition and signal extraction apparatus as informational rent, which takes the form of a 'quasi-option'. Section 3 develops results for general quasi-option sequences, including the value of such sequences, the form and properties of intertemporally optimal signal extraction policies (section 3.1), and the observational indistinguishability between short-termism and such intertemporally optimal policies (section 3.2). Section 4 concludes. Longer mathematical derivation and proofs are collected in the appendix.

2 Preliminaries

2.1 Two-period model

Let us initially consider the following simplified two-period scheme. The economy consists of entrepreneurs, relationship intermediaries and arm's-length financiers. All are assumed to be risk-neutral. Each entrepreneur has a business plan for a two-period project¹ that requires an irreversible investment of K_{it} at the beginning of each period. The entrepreneur has no capital of her own save human capital, but since there is a complementary relationship between the latter and the 'live' project, the entrepreneur always has incentives to carry the project to its full term. At the end of each period, project i repays B_{it} with probability $(1-p_i)$ and defaults with probability p_i , repaying nothing. Thus p_i is a measure of project risk. The project may be continued into the second period only if it is a 'success' in the first period, i.e., only if the agreed repayment of B_{i1} is made. Project risk p_i and loan repayments B_{it} are fully observable by both inside and outside financiers. A project's required rate of return is competitively determined and reflects publicly known project risk p_i . The B_{it} may be viewed as the face value repayable on a pure discount instrument given competitive pricing, p_i and K_i .

In addition to limiting the present work-up model to two periods, the following simplifications

¹The first period begins at time $t = 0$ and ends at time $t = 1$; the second period begins at time $t = 1$ and ends at time $t = 2$.

will also be put in place. Firstly, we abstract from the time value of money. Accordingly, the risk-free rate of interest may be set to zero, thereby simplifying investment requirements $K_{i0} = K_{i1} = K_i$, repayments $B_{i1} = B_{i2} = B_i$ and the relationship between the two. Time subscripts will be dropped here where no ambiguity results from doing so. Secondly, assuming that projects are only subject to technical or other forms of idiosyncratic risk that are uncorrelated with market returns, returns need not be adjusted to incorporate compensation for systematic risk. In short, the discount factor is simply unity. As applied to the competitive market pricing of the risk–return trade-off, these considerations imply that the repayment B_{it} that the relationship intermediary can successfully secure for any given project risk class p is only just sufficient to ensure a zero net present value using public information, i.e., $(1-p_i)B_i = K_i$.

2.2 Information

In the literature, relationship intermediaries are often portrayed as having specialised skills for eliciting and interpreting soft information and subtle signals (Bhattacharya and Thakor, 1993; Berger and Udell, 2002). Yet these portrayals stop short of formalising of the precise nature of such soft information, subtle signals and specialised skills.

The simplest possible representation of informational rent is premised on opaqueness. The relationship intermediary gains access to a noisy signal of the client firm’s next-period performance while to arm’s-length financiers, the firm remains opaque. Arm’s-length financiers can observe neither this signal nor the details from which this signal could be deduced. The relationship intermediary’s informational rent, then, is equal to the increase in expected net present value obtained by conditioning the funding decision on this noisy signal. As Conrad (1980) first pointed out, the value of information measured in this fashion corresponds to the Arrow-Fisher-Hanemann-Henry notion of quasi-option value (Arrow and Fisher, 1974; Hanemann, 1989; Henry, 1974).

By positing a refinement of the information acquisition apparatus, we attempt to go beyond both the simple opaqueness premise of informational rent as well as the general notions of soft information invoked in the literature. The present study recognises that firms may divulge information via both public channels as well as private channels. The first defining characteristic of a relationship intermediary’s information acquisition apparatus is the existence of a private channel of communication with the client firm. Through this channel, protected by commer-

cial confidentiality clauses, the relationship intermediary gains access to ‘inside’ information. This private channel remains opaque to arm’s-length financiers. The second defining characteristic concerns the nature of information transmitted. The relationship intermediary builds up, through repeated interactions with the firm, not only information about track record and credit risk – i.e. ‘type’ information² – but also detailed information about the firm, its context and operations. By cross-referencing with this knowledge base and deriving the logical implications, the relationship intermediary may reduce the residual ambiguity in its interpretation of subsequent messages. Both the type assessment and the knowledge base are updated over time. But whereas the former may be inferred, if only coarsely, by arm’s-length financiers from observing the relationship intermediary’s history of funding the firm,³ the latter is available only to the relationship intermediary – it resides, in proprietary embodied form, with the relevant loan or investment officer. It is the conjunction of the private communication channel with this knowledge base of detailed particularistic information that allows the relationship intermediary to extract a signal regarding the firm’s next-period performance.

2.3 Signal extraction

The signal is extracted as a binary classifier for whether the project will prove a success or a failure in the coming period. The relationship intermediary’s signal detection mechanism augments public information $\mathcal{I}^{\text{pu}}$ with the private knowledge base $\mathcal{I}_i^{\text{kb}}$.

Following the standard signal detection theory formulation, the problem reduces to the determination of an optimal cutoff threshold $\theta^* \in \Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$ that identifies the observed score $\theta = \Gamma(\mathcal{I})$ as belonging either to the interval associated with success ($\underline{\theta} \leq \theta \leq \theta^*$) or with failure ($\theta^* < \theta \leq \bar{\theta}$) by applying an optimality criterion to the Receiver Operating Characteristics (ROC) curve⁴ (Green and Swets, 1966; Egan, 1975; Macmillan and Creelman, 1991).

Arm’s-length investors may implement the scoring procedure $\Gamma_i(\mathcal{I}^{\text{pu}})$ purely as a statistical model, and indeed even relationship intermediaries do use the results of statistical models drawing on public information and private institutional information gathered from across client relationship portfolios. Crucially, however, the investment officer responsible for firm i brings

²see e.g. Diamond (1991)

³see e.g. Lummer and McConnell (1989)

⁴This curve “describes the inherent detection *characteristics* of the test....since the *receiver* of the test information can *operate* at any point on the curve by using an appropriate decision threshold” (Metz, 1978).

$\mathcal{I}_i^{\text{kb}}$ and her own analytical skills to bear on the case-specific validity of these results. In other words, it is the particularistic features and characteristics of i represented in $\mathcal{I}_i^{\text{kb}}$ and the extent to which the investment officer brings this to bear on the scoring procedure that distinguishes $\Gamma_i(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})$. Not all details present in $\mathcal{I}_i^{\text{kb}}$ are necessarily relevant to the scoring procedure. However some will be – although perhaps not evident without in-depth examination – and the performance of $\Gamma_i(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})$ depends on how fully it reflects the implications of such details. For concreteness, an example of how the investment officer’s private information may be thought to be brought to bear on the scoring procedure is presented in section 2.3.1.

The success of a scoring procedure $\Gamma(\mathcal{I})$ falls between perfect discrimination ($\exists \theta' : P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{fail}) = 1, P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{succ}) = 0$) and pure chance ($P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{fail}) = P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{succ}) \forall \theta' \in \Theta$). For every scoring procedure $\Gamma(\mathcal{I})$ each particular threshold value θ' defines a combination of True Negative Likelihood (TNL), False Positive Likelihood (FPL), True Positive Likelihood (TPL) and False Negative Likelihood (FNL), where the former pair and the latter pair are complementary ($\text{FPL}_{\theta', \Gamma(\mathcal{I})} = 1 - \text{TNL}_{\theta', \Gamma(\mathcal{I})}$ and $\text{TPL}_{\theta', \Gamma(\mathcal{I})} = 1 - \text{FNL}_{\theta', \Gamma(\mathcal{I})}$). Given that $H_0 : \theta_{\Gamma(\mathcal{I})} \leq \theta'$ (no signal of failure imminent) while $H_1 : \theta_{\Gamma(\mathcal{I})} > \theta'$ (signal ‘failure imminent’), the following correspondences hold:

$$\begin{aligned} \text{TNL}_{\theta', \Gamma(\mathcal{I})} &= P(\text{nosig}_{\theta', \Gamma(\mathcal{I})} | \text{succ}) = P(\theta_{\Gamma(\mathcal{I})} \leq \theta' | \text{succ}) = (1 - \alpha_{\theta', \Gamma(\mathcal{I})}) = \text{Specificity} \\ \text{FPL}_{\theta', \Gamma(\mathcal{I})} &= P(\text{sig}_{\theta', \Gamma(\mathcal{I})} | \text{succ}) = P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{succ}) = \alpha_{\theta', \Gamma(\mathcal{I})} = \text{Type I error likelihood} \\ \text{TPL}_{\theta', \Gamma(\mathcal{I})} &= P(\text{sig}_{\theta', \Gamma(\mathcal{I})} | \text{fail}) = P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{fail}) = (1 - \beta_{\theta', \Gamma(\mathcal{I})}) = \text{Power} \\ \text{FNL}_{\theta', \Gamma(\mathcal{I})} &= P(\text{nosig}_{\theta', \Gamma(\mathcal{I})} | \text{fail}) = P(\theta_{\Gamma(\mathcal{I})} \leq \theta' | \text{fail}) = \beta_{\theta', \Gamma(\mathcal{I})} = \text{Type II error likelihood} \end{aligned}$$

In turn, the ROC curve for a scoring procedure $\Gamma(\mathcal{I})$ plots the $\text{TPL}_{\theta', \Gamma(\mathcal{I})}$ on the vertical axis of the unit square against the $\text{FPL}_{\theta', \Gamma(\mathcal{I})}$ on the horizontal axis of the unit square as the threshold θ' is varied within its domain. The ROC curve for $\Gamma(\mathcal{I})$ consists of the locus of points

$$\{ (P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{succ}), P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{fail})) : \theta' \in \Theta \}. \quad (2.1)$$

This curve is positively sloped, running from the bottom left-hand corner to the top right-hand corner of ROC space. The major diagonal connecting these two corners is the chance line ($P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{fail}) = P(\theta_{\Gamma(\mathcal{I})} > \theta' | \text{succ}) \forall \theta' \in \Theta$). The Area Under the Curve (AUC) in ROC space is a measure of the probability that the scoring procedure will rank a randomly chosen failure more highly than a randomly chosen success.⁵ Hence scoring procedures with ROC curves

⁵The AUC is equivalent to the Wilcoxon-Mann-Whitney test.

that coincide with the major diagonal have $AUC = \frac{1}{2}$ and do no better than categorisation by randomisation, while scoring procedures that achieve perfect discrimination have $AUC = 1$.

The AUC serves as a summary measure of an agent's information acquisition and processing performance. A relationship intermediary's scoring procedure $\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})$ improves upon the arm's-length investor's scoring procedure $\Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})$ in the sense that $AUC(\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})) > AUC(\Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}}))$. Meanwhile, the firm's management itself will do at least as well as the relationship intermediary, but still not without some strictly positive residual likelihood of Type-I and Type-II error $1 > AUC(\Gamma_i^{\text{M}}(\mathcal{I}^{\text{M}})) \geq AUC(\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}}))$. Hence a natural candidate measure of the *closeness* of the relationship is $\psi = \frac{AUC(\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})) - \frac{1}{2}}{AUC(\Gamma_i^{\text{M}}(\mathcal{I}^{\text{M}})) - \frac{1}{2}}$ for all interesting cases where $AUC(\Gamma_i^{\text{M}}(\mathcal{I}^{\text{M}})) \geq AUC(\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})) > \frac{1}{2}$. Thus $\psi : (\frac{1}{2}, 1) \times (\frac{1}{2}, 1) \rightarrow [0, 1]$ is a measure of the degree to which the relationship lender is truly an 'insider' – it measures the relationship lender's proximity to management and its access to internal managerial information. Consistent with the findings of the empirical relationship lending literature, it is reasonable that the evolution of ψ_t over time will be positively correlated with the duration of the relationship (Berger and Udell, 1995), possibly extending over a sequence of several distinct projects, as well as the degree of mutual trust in the relationship (Harhoff and Körting, 1998). Nevertheless throughout this paper it will be assumed that the relationship intermediary's ROC curve, and hence the AUC, remains fixed over the lifetime of project i .

insert Figure 1 (see p. 22) about here

Denoting the direct cost of implementing the generic scoring procedure as $C_{\Gamma(\mathcal{I})}$ and the costs associated with true positives, false negatives, true negatives and false positives as C_{TP} , C_{FN} , C_{TN} and C_{FP} respectively, then the expected cost of using the signal extraction mechanism based on $\Gamma(\mathcal{I})$ is of the form

$$\begin{aligned}
E(C) &= C_{\Gamma(\mathcal{I})} + C_{\text{TP}}P(\text{TP}) + C_{\text{FN}}P(\text{FN}) + C_{\text{TN}}P(\text{TN}) + C_{\text{FP}}P(\text{FP}) \\
&= -[C_{\text{FN}} - C_{\text{TP}}]P(\text{fail})\text{TPL} + [C_{\text{FP}} - C_{\text{TN}}]P(\text{succ})\text{FPL} \\
&\quad + C_{\Gamma(\mathcal{I})} + C_{\text{TN}}P(\text{succ}) + C_{\text{FN}}P(\text{fail}) \quad .
\end{aligned} \tag{2.2}$$

ROC curves are continuous, but need not be differentiable. For ROC curves that are differentiable, θ^* identifies the point $(\text{FPL}_{\theta^*, \Gamma(\mathcal{I})}, \text{TPL}_{\theta^*, \Gamma(\mathcal{I})})$ at which the isocost line is tangent to the

ROC $_{\Gamma(\mathcal{I})}$ curve.⁶ Yet here we also wish to allow for piecewise linear ROC curves that are not differentiable. That is, we wish to minimise the expected costs of implementing the decision criterion $E(C)$ subject to the TPL and FPL parameters being constrained by the (possibly not differentiable) ROC curve generated by $\Gamma(\mathcal{I})$.

$$\min E(C) \quad \text{s.t.} \quad \text{TPL} = G_{\Gamma(\mathcal{I})}^{\text{ROC}}(\text{FPL}) \quad (2.3)$$

From setting the total differential of expected cost to zero

$$dE(C) = -[C_{\text{FN}} - C_{\text{TP}}]P(\text{fail}) d\text{TPL} + [C_{\text{FP}} - C_{\text{TN}}]P(\text{succ}) d\text{FPL} = 0 \quad (2.4)$$

it follows that the slope of each isocost line – and therefore also the slope of the cost minimising isocost line at the optimal operating point – is the probability weighted ratio of the cost of success to the cost of failure

$$\left(\frac{d\text{TPL}}{d\text{FPL}}\right)_{\bar{C}} = \frac{P(\text{succ})}{P(\text{fail})} \frac{[C_{\text{FP}} - C_{\text{TN}}]}{[C_{\text{FN}} - C_{\text{TP}}]} = \left(\frac{d\text{TPL}}{d\text{FPL}}\right)_{\bar{C}^*} . \quad (2.5)$$

2.3.1 Putting private information to use

In the previous section the relationship intermediary’s signal extraction was discussed in general and broad terms, leaving as unspecified the details of how the investment officer may bring her knowledge base and analytical skills to bear on the scoring procedure. Here we present a simple specific example of how this may be formalised analytically by adapting Luce’s (1963) two-state threshold model (also see Green and Swets, 1966, p. 71–74).⁷

The investment officer has at her disposal the results of the arm’s-length scoring procedure $\Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})$ as a starting point.⁸ As this scoring procedure yields no information not already incorporated into the publicly known success $(1-p_i)$ and failure p_i probabilities, the ROC $_{\Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})}$ curve is simply the chance line joining $(0, 0)$ and $(1, 1)$ in ROC space. If the relationship intermediary were to proceed on the basis of this information alone,⁹ equation (2.5) would prove insufficient

⁶See appendix A.1 for a direct derivation of the optimal operating point.

⁷Luce’s (1963) two-state model has come to be known as the ‘low-threshold model’, to distinguish it from the empirically less successful ‘single high-threshold theory’ and the ‘double high-threshold theory’ (Macmillan and Creelman, 1991).

⁸We could assume that the investment officer takes as her starting point the relationship intermediary’s internal statistical scoring procedure which exploits cross-product and cross-market information. Nevertheless we cannot discern an appreciable enhancement of our analysis from doing so.

⁹yielding zero informational rent

to uniquely identify a critical threshold $\theta'_{\Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})}$, and barring mixed strategies (randomisation) some other criterion would have to be adduced, such as the Neyman-Pearson Lemma given some specification of the organisationally acceptable false positive likelihood. Thus, prior to the incorporation of the investment officer's private knowledge base, the associated $\text{FNL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})}$ and $\text{TNL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})}$ are

$$\text{FNL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})} = \int_{\underline{\theta}}^{\theta'} f(\theta | \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}}), \text{fail}) d\theta = \text{TNL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})} = \int_{\underline{\theta}}^{\theta'} f(\theta | \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}}), \text{succ}) d\theta \quad (2.6a)$$

while the $\text{TPL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})}$ and $\text{FPL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})}$ are

$$\text{TPL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})} = \int_{\theta'}^{\bar{\theta}} f(\theta | \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}}), \text{fail}) d\theta = \text{FPL}_{\theta', \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}})} = \int_{\theta'}^{\bar{\theta}} f(\theta | \Gamma_i^{\text{AL}}(\mathcal{I}^{\text{pu}}), \text{succ}) d\theta. \quad (2.6b)$$

The investment officer reconsiders these likelihoods in light of her knowledge base $\mathcal{I}_i^{\text{kb}}$ and analytical capabilities $\Gamma_i^{\text{RI}}(\cdot, \cdot)$. This reconsideration consists of the elimination of logical inconsistencies, errors and ambiguity given detailed particularistic knowledge of the firm and its project $\mathcal{I}_i^{\text{kb}}$, and results in adjustments to the FNL, TNL, TPL and FPL over those derived from the public information scoring procedure (2.6ab).

$$\begin{aligned} \text{FNL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})} &= \int_{\underline{\theta}}^{\theta'} f(\theta | \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}}), \text{fail}) d\theta \\ &< \text{TNL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})} = \int_{\underline{\theta}}^{\theta'} f(\theta | \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}}), \text{succ}) d\theta \end{aligned} \quad (2.7a)$$

$$\begin{aligned} \text{TPL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})} &= \int_{\theta'}^{\bar{\theta}} f(\theta | \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}}), \text{fail}) d\theta \\ &> \text{FPL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})} = \int_{\theta'}^{\bar{\theta}} f(\theta | \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}}), \text{succ}) d\theta \end{aligned} \quad (2.7b)$$

This also affects the ROC curve, for the $\text{ROC}_{\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})}$ curve associated with (2.7ab) consists of two line segments: a lower leg from $(0, 0)$ to $(\text{TPL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})}, \text{FPL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})})$, and an upper leg from $(\text{TPL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})}, \text{FPL}_{\theta', \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})})$ to $(1, 1)$. Here $\text{AUC}_{\Gamma_i^{\text{RI}}} > \frac{1}{2}$, making strictly positive informational rent possible. Such a kinked ROC curve is illustrated in Figure 2.

The transformation of (2.6ab) into (2.7ab) is a representation of the investment officer's cognitive processes: a subjective, though not rigour-free recalibration of (2.6ab) in light of her knowledge base $\mathcal{I}_i^{\text{kb}}$ and analytical capabilities $\Gamma_i^{\text{RI}}(\cdot, \cdot)$. The results of this process could, in principle, be formalised and documented mathematically by the investment officer. In this sense,

(2.7ab) need not remain ‘soft information’ as Petersen (2002) defines it. However in most cases the opportunity costs of doing so are likely to be prohibitive.

insert Figure 2 (see p. 23) about here

2.4 Informational rent

Relative to arm’s-length financiers, the signal extracted by relationship intermediaries permits a more active and less wasteful funding policy. It proves convenient to introduce the parameter ε_i , defined as the conditional probability that, at the end of the first period but prior to the commitment of K_i at the beginning of the second period, the relationship intermediary learns that the project i will fail in the subsequent (second) period. This is the TPL that the relationship intermediary receives the signal ‘failure imminent’, i.e., $\varepsilon_i = P(\text{sig}|\text{fail}) = \text{TPL} = (1 - \beta)$.

The increase in expected net present value over that obtained with public information – achieved by conditioning the funding decision on the noisy signal ($\varepsilon_i, \alpha_i > 0$) – simplifies to the following compact expression:

$$E(\text{NPV} | \varepsilon_i, \alpha_i > 0) - E(\text{NPV} | \varepsilon_i, \alpha_i = 0) = (1 - p_i)p_i(\varepsilon_i - \alpha_i)K_i . \quad (2.8)$$

This increase in expected NPV is the expected value of information generated by the relationship intermediary’s information acquisition and signal extraction apparatus. The operation of this apparatus endows the relationship intermediary with a ‘quasi-option’, the value of which is equivalent to the expected value of information generated by this apparatus (Conrad, 1980). Thus ‘the value of information’, ‘quasi-option value’ and ‘informational rent’ are equivalent and interchangeable labels for (2.8).

Intuitively, expression (2.8) represents the expected fraction of the irreversibly committed principal K_{i1} that is saved through the suspension of projects that give the signal ‘failure imminent’ upon successfully reaching the end of the first period. It is non-negative for reasonable parameter values. Direct inspection reveals that this informational rent (2.8) is increasing in the true positive likelihood ε_i and the magnitude of the irreversible (second period) investment K_{i1} , but decreasing in the false positive likelihood α_i . Since $(\varepsilon_i - \alpha_i)$ is equivalent to the Youden (1950) index $J = \text{TPL} + \text{TNL} - 1$ and both α_i and ε_i are constrained by the $\text{ROC}_{\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})}$

curve trade-off, we can see that (2.8) is maximised by maximising the Youden index J subject to the constraints imposed on α_i and ε_i by $\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kbi}})$. The equal weighting of α_i and ε_i here in this two-period model, where informational rent arises only in the final period, is also evident from the unitary slope that the isocost line (2.5) takes at the optimal operating point.¹⁰

Informational rent (2.8) may be seen as an expression of the ‘bad news principle’, whereby (quasi-) option value is an increasing function of the magnitude of bad outcomes (here, the loss of the principal K_{i1}) and the probability with which these bad outcomes are realised. It is precisely these bad outcomes that are avoided through optimal exercise of the quasi-option giving the right, but not the obligation, to make the second period investment K_{i1} .

3 General quasi-option sequence

In order to state precisely the trade-off between proximate and future informational rent it is necessary to extend the model beyond two periods.¹¹ It is here that we investigate the form of optimal signal extraction within a multi-period elaboration of the previous work-up model of informational rent extraction.

The general value expression for the relationship intermediary’s quasi-option sequence (informational rent; value of information) starting at time τ and ending at the finite time T_i has

¹⁰This may be confirmed by substituting $C_{\text{TP}} = -K_i$, $C_{\text{FN}} = K_i$, $C_{\text{TN}} = -(B_i - K_i)$ and $C_{\text{FP}} = (B_i - K_i)$ into (2.5).

¹¹Three periods would suffice for some purposes, but we develop the model for arbitrary finite project i lifetime $T_i \in \mathbb{N} \setminus \{1\}$

the following form.

$$\begin{aligned}
& \mathcal{Q}_i(\tau, T_i, K_i, B_i, p_i, \boldsymbol{\varepsilon}_i, \boldsymbol{\alpha}_i, \delta_i, \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}^{\text{kb}})) \\
&= \delta_i^{\mathcal{H}(\tau)} (1-p_i)^{\mathcal{H}(\tau)} p_i K_i \sum_{t=\mathcal{H}(\tau)+\tau}^{T_i-1} (\varepsilon_{it} - \alpha_{it}) \left[\delta_i (1-p_i) \{p_i(1-\varepsilon_{it}) + (1-p_i)(1-\alpha_{it})\} \right]^{t-\mathcal{H}(\tau)-\tau} \\
&\quad \tau \in (0, 1, \dots, T_i-1), \quad T_i \in \mathbb{N} \setminus \{1\} \\
&\quad \boldsymbol{\varepsilon}_i = (\varepsilon_{i(\mathcal{H}(\tau)+\tau)}, \varepsilon_{i(\mathcal{H}(\tau)+\tau+1)}, \dots, \varepsilon_{i(T_i-1)}) \\
&\quad \boldsymbol{\alpha}_i = (\alpha_{i(\mathcal{H}(\tau)+\tau)}, \alpha_{i(\mathcal{H}(\tau)+\tau+1)}, \dots, \alpha_{i(T_i-1)}) \\
&\quad \mathcal{H}(\tau) = 2H(-\tau) = \begin{cases} 1 & \tau = 0 \\ 0 & \tau \geq 1 \end{cases} \quad (3.1) \\
&\quad \delta_i = \frac{1}{1 + r_f + \phi \sigma_{im}} \quad \begin{aligned} \phi &= (r_m - r_f) / \sigma_m^2 \\ \sigma_{im} &= \text{Cov}(r_i, r_m) \end{aligned} \\
&\quad K_{it} = K_{it+1} \quad \forall t, \quad K_{it} = \delta_i (1-p_i) B_{it+1} \quad \forall t
\end{aligned}$$

This formulation presupposes that the relationship intermediary's scoring technology $\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}^{\text{kb}})$ and hence its $\text{ROC}_{\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}^{\text{kb}})}$ curve does not change during the life of project i . Likewise, the parameters K_i , B_i , p_i and δ_i are taken to be common to all periods. But the true positive and false positive likelihoods are period-specific; hence the $(T_i - \mathcal{H}(\tau) - \tau)$ -dimensionality of the vectors $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\alpha}_i$. In turn $\mathcal{H}(\tau)$ is defined as a transformation of the Heaviside function $H(\cdot)$ to yield the value 1 for $\tau=0$ and the value 0 for $\tau \geq 1$. It is used here to account for the presence or absence of period 1, during which no informational rent is possible. δ_i may be recognised as the standard risk-adjusted discount factor with risk free rate r_f and market price of risk ϕ . Finally, note that the term in braces is equivalent to $P_{it}(\text{nosig})$.

3.1 Intertemporally optimal signal extraction

The value of (3.1) is maximised for an appropriate specification of $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\alpha}_i$, namely the optimal values $\boldsymbol{\varepsilon}_i^* = (\varepsilon_{i(\mathcal{H}(\tau)+\tau)}^*, \varepsilon_{i(\mathcal{H}(\tau)+\tau+1)}^*, \dots, \varepsilon_{i(T_i-1)}^*)$ and $\boldsymbol{\alpha}_i^* = (\alpha_{i(\mathcal{H}(\tau)+\tau)}^*, \alpha_{i(\mathcal{H}(\tau)+\tau+1)}^*, \dots, \alpha_{i(T_i-1)}^*)$. These depend on the scoring technology $\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}^{\text{kb}})$ as well as on probability weighted ratio of the cost of success to the cost of failure, as shown in (2.5). But because of the sequential dependence of investment opportunities over time, the cost of false positives $C_{\text{FP}_{is}}$ and true negatives $C_{\text{TN}_{is}}$ at time s are a function of the number of investment periods remaining in the project's potential lifetime. Given an ROC curve, these costs $C_{\text{FP}_{is}}, C_{\text{TN}_{is}}$ and hence the optimal true and false positive likelihoods $\varepsilon_{is}^*, \alpha_{is}^*$ may be uniquely identified recursively. The

four respective costs for the time $s \in (1, \dots, T_i - 1)$ investment decision may be written in terms of $\delta_i B_{is+1}$, K_{is} and $\delta_i Q_i(s+1, T_i, \epsilon_i^*, \alpha_i^*)$, where the latter is the time s value of the quasi-option sequence from time $s+1$ onward.¹²

$$\begin{aligned}
C_{FP_{is}} &= \delta_i B_{is+1} - K_{is} + \delta_i Q_i(s+1, T_i, \epsilon_i^*, \alpha_i^*) \\
C_{TN_{is}} &= -\{\delta_i B_{is+1} - K_{is} + \delta_i Q_i(s+1, T_i, \epsilon_i^*, \alpha_i^*)\} = -C_{FP_{is}} & s \in (1, \dots, T_i - 1) \\
C_{FN_{is}} &= K_{is} & s = T - 1 \Rightarrow \\
C_{TP_{is}} &= -K_{is} = -C_{FN_{is}} & Q_i(s+1) = 0
\end{aligned} \tag{3.2}$$

Recall that $\left(\frac{d\epsilon_{iT_i-1}}{d\alpha_{iT_i-1}}\right)_{\bar{C}^*} = 1$ was obtained in section 2.4. From (3.2) we see that $Q_i(s+1) = 0$ for $s = T_i - 1$ and hence that indeed $\left(\frac{d\epsilon_{iT_i-1}}{d\alpha_{iT_i-1}}\right)_{\bar{C}^*} = \frac{(1-p_i)}{p_i} \frac{2(\delta_i B_i - K_i)}{2K_i} = \frac{(1-p_i)}{p_i} \frac{(\delta_i K_i / \delta_i (1-p_i) - K_i)}{K_i} = \frac{(1-p_i)}{p_i} \frac{\delta_i K_i - \delta_i (1-p_i) K_i}{\delta_i (1-p_i) K_i} = \frac{p_i}{p_i} = 1$. For $s \in (1, 2, \dots, T-1)$ arbitrary,

$$\left(\frac{d\epsilon_{is}}{d\alpha_{is}}\right)_{\bar{C}^*} = 1 + \sum_{t=s+1}^{T_i-1} \delta_i^{t-s} (1-p_i)^{t-s} (\epsilon_{it}^* - \alpha_{it}^*) \{p_i(1-\epsilon_{it}^*) + (1-p_i)(1-\alpha_{it}^*)\}^{t-s-1} . \tag{3.3}$$

At the time s optimal operating point $(\epsilon_{is}^*, \alpha_{is}^*)$, the derivative of the relationship intermediary's ROC curve, assuming that this derivative is defined at all, must be equal to the slope of the expected cost minimising isocost line $\left(\frac{d\epsilon_{is}}{d\alpha_{is}}\right)_{\bar{C}^*}$. This slope is nondecreasing in the number of investment periods remaining $T_i - s$. For all ROC curves (scoring technologies Γ_i^{RI} and knowledge bases $\mathcal{I}_i^{\text{kb}}$),

$$(\epsilon_{i1}^* - \alpha_{i1}^*) \leq (\epsilon_{i2}^* - \alpha_{i2}^*) \leq \dots \leq (\epsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) . \tag{3.4}$$

Yet for the equation (3.3) term in braces $p_i(1-\epsilon_{it}^*) + (1-p_i)(1-\alpha_{it}^*) = 1 - p_i\epsilon_{it}^* - (1-p_i)\alpha_{it}^* = P_{it}(\text{nosig})$ the opposite holds

$$\begin{aligned}
1 - p_i\epsilon_{iT_i-1}^* - (1-p_i)\alpha_{iT_i-1}^* &\leq 1 - p_i\epsilon_{iT_i-2}^* - (1-p_i)\alpha_{iT_i-2}^* \leq \dots \leq 1 - p_i\epsilon_{i1}^* - (1-p_i)\alpha_{i1}^* \\
P_{iT_i-1}^*(\text{nosig}) &\leq P_{iT_i-2}^*(\text{nosig}) \leq \dots \leq P_{i1}^*(\text{nosig}) .
\end{aligned} \tag{3.5a}$$

This means that as the relationship-to-go is longer, the relationship intermediary optimally sets the probability of eliciting the signal 'failure imminent' no higher than if the relationship-to-go were shorter. All else being equal, the longer the relationship-to-go, the greater the expected future value of informational rents prior to discounting. But the impact of these future informational rents on the current-period decision is transmitted via (3.3) applied to the ROC curve. Hence more specific conclusions – about the optimal operating point $(\epsilon_{is}^*, \alpha_{is}^*)$,

¹²For economy of presentation, this expression for Q suppresses parameters that are well-specified in (3.1).

its evolution with relationship-to-go $T_i - s$, and the value of the relationship intermediary's informational rent $Q_i(\tau, T_i)$ – hinge on the shape of the $\text{ROC}_{\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})}$ curve induced by the scoring procedure $\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})$. The same dependence holds for the evolution of $P_{it}(\text{nosig})$ over time.

3.1.1 Differentiable ROC curves

In the case that $\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})$ generates a binormal distribution in θ ,¹³ the associated $\text{ROC}_{\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})}$ curve will be strictly concave and symmetric about the minor diagonal. Under this restriction the optimal final period operating point $(\varepsilon_{iT_i-1}^*, \alpha_{iT_i-1}^*)$ coincides with the intersection of the $\text{ROC}_{\Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}})}$ curve and the minor diagonal, and because $\frac{d\varepsilon_{is}^*}{d\alpha_{is}^*} > \frac{d\varepsilon_{is+1}^*}{d\alpha_{is+1}^*}$ the optimal operating points of earlier periods are shifted to the south-west from those of later periods, $(\varepsilon_{is}^* < \varepsilon_{is+1}^*; \alpha_{is}^* < \alpha_{is+1}^*)$. Hence under binormality of θ , (3.5a) becomes

$$P_{iT_i-1}^*(\text{nosig}) < P_{iT_i-2}^*(\text{nosig}) < \dots < P_{i1}^*(\text{nosig}) , \quad (3.5b)$$

in which case it is optimal for the relationship intermediary to be more lenient in judging the prospects for next-period success the longer the relationship-to-go. (See Figure 3a.)

3.1.2 Non-differentiable ROC curves

For irregular ROC curves with kinks, such as the piecewise linear ROC curve motivated in section 2.3.1, the lowest isocost line achievable is that which is furthest to the north-west and still coincides, at one point at least, with the ROC curve. However as a consequence of this kink, application of (3.3) yields $\varepsilon_{is}^* = \varepsilon_{iv}^*$, $\alpha_{is}^* = \alpha_{iv}^* \forall s, v \in U$, $s \neq v$ for some non-degenerate interval U of $(1, 2, \dots, T_i - 1)$. For some parameter constellations it will indeed hold that

$$P_{iT_i-1}^*(\text{nosig}) = P_{iT_i-2}^*(\text{nosig}) = \dots = P_{i1}^*(\text{nosig}) . \quad (3.5c)$$

Therefore in cases where the ROC curve is kinked, the optimal investment policy may be insensitive to changes in K_i and T_i – and thereby observationally indistinguishable from short-termism – over some interval U , if not throughout the project's entire duration. (See Figure 3b.) Where (3.5c) holds, the value of the relationship intermediary's informational rent simplifies to

$$\delta_i^{\mathcal{H}(\tau)} (1-p_i)^{\mathcal{H}(\tau)} p_i (\varepsilon_i^* - \alpha_i^*) K_i \sum_{t=\mathcal{H}(\tau)+\tau}^{T_i-1} \left[\delta_i (1-p_i) \{ p_i (1-\varepsilon_i^*) + (1-p_i) (1-\alpha_i^*) \} \right]^{t-\mathcal{H}(\tau)-\tau} . \quad (3.6)$$

¹³that is $f(\theta|\text{succ}) \sim N(m_s, \sigma_s^2)$, $f(\theta|\text{fail}) \sim N(m_f, \sigma_f^2)$, where $m_s < m_f$ and $\sigma_f^2 = \sigma_s^2$

insert Figure 3 (see p. 24) about here

3.2 Short-termism and observational indistinguishability

The definition of short termism cited in the introduction is prevalent in the public sphere. In a more strict economic sense, short-termism may be defined as “excess discounting of expected cash flows that accrue further in the future” (Miles, 1995). Some authors would wish to maintain a distinction between short-termism and investor myopia, where the latter is reserved for investors who are institutionally constrained, or incentivised through performance evaluation, to invest over a comparatively short time horizon. Technically however the investor’s horizon will be limited to t periods if for all subsequent periods $t' > t$ the discount rate rises without bound and $\delta_{t'} \rightarrow 0$. Since discounting at an infinite rate is one particular example of excessive discounting, investor myopia – that is, an excessively short planning horizon – is therefore but one possible manifestation of short-termism.

In the present model, the relationship intermediary employs a constant discount rate that is set to correctly account for the opportunity cost of capital. Here different investment policies result not from varying the discount rate, but from different specifications of the information acquisition apparatus and signal extraction mechanism – which jointly determine the shape of the ROC curve – and the costs of false negatives, true positives, false positives and true negatives.

Yet onlookers do not directly observe these details. Outsiders can only observe parameters in the public domain, including (a) the project’s risk p_i , (b) the project’s funding requirements K_i , (c) the project’s potential lifetime T_i , and (d) the reasons the relationship intermediary has given for approving finance to project i up to time $t-1$ and then for refusing it at time t , either (i) $(\theta_{i\tau} \leq \theta_{i\tau}^* \forall \tau \in \{1, 2, \dots, t-1\})$ and $\theta_{it}^* < \theta_{it}$ or (ii) $\text{NPV}_{i\tau} \geq 0 \forall \tau \in \{1, 2, \dots, t-1\})$ and $\text{NPV}_{it} < 0$. Operating within the $\{a, b, c, d, i\}$ information set, the outside observer expects θ_i^* to respond to changes in K_i and T_i as $\frac{d\theta_i^*(K_i, T_i)}{dK_i} > 0$ and $\frac{d\theta_i^*(K_i, T_i)}{dT_i} > 0$ in order to reflect increased informational rents accruing to the relationship intermediary when either K_i or T_i increases. If the outsider observes either $\theta_{it}^*(K_{it}, T_i) = \theta_{it}^*(K_{it-1} + dK, T_i) = \theta_{it-1}^*(K_{it-1}, T_i)$ for $dK > 0$ or $\theta_{it}^*(K_i, T_{it}) = \theta_{it}^*(K_i, T_{it-1} + dT) = \theta_{it-1}^*(K_i, T_{it-1})$ for $dT > 0$, he concludes erroneously that the relationship intermediary must be excessively discounting future payouts, and that therefore

the relationship intermediary's funding policy displays an evident short-termist bias. When the outsider is limited to the {a,b,c,d.ii} information set and does not observe the full probability distributions updated to reflect private signal information, then similarly, 'excessive discounting' is imputed as the cause for the discontinuation of funding at time t .

With the benefit of an explicit model of signal extraction optimised to maximise informational rents it becomes clear that the discount rate is only one consideration in striking the optimal trade-off between proximate and more distant payouts. At the same time, it also becomes clear that unanticipated changes to certain parameters, in particular the investment principal K_i and the project's lifetime T_i , do not necessarily lead to a change in the intertemporally optimal investment policy.

From the time τ value of the quasi-option sequence $\mathcal{Q}_i(\tau, T_i, K_i, B_i, p_i, \varepsilon_i, \alpha_i, \delta_i, \Gamma_i^{\text{RI}}(\mathcal{I}^{\text{pu}}, \mathcal{I}_i^{\text{kb}}))$ specified in equation (3.1) it is evident that the relationship intermediary's informational rents are a fractional multiple of the investment principal K_i . A perturbation that permanently increases the recurring investment requirement from time τ onward therefore also increases the value of the relationship intermediary's informational rents. However, because the slope of the isocost line $\left(\frac{d\varepsilon_{i\tau}}{d\alpha_{i\tau}}\right)_{\bar{C}^*}$ is equal to the probability weighted ratio of the cost of success to the cost of failure, both of which are simply multiples of the principal K_i , the level of the principal K_i itself cancels out of the expression for the optimal operating point and therefore does not influence the optimal investment policy. This result is a direct consequence of (3.3). We condense and label it for future reference as follows.

Lemma 3.1 (Stake independence). The optimal funding policy, formalised in the threshold $\theta_{i\tau}^*$ that determines the optimal operating point $(\alpha_{i\tau}^*, \varepsilon_{i\tau}^*)$ for each $\tau \in (1, 2, \dots, T_i - 1)$, is independent of contemporaneous, permanent upward or downward perturbation of the principal K_i . This holds for all ROC curves and for all finite perturbation magnitudes.

Importantly, stake independence holds not only for differentiable ROC curves, but also kinked ROC curves. Therefore regardless of the specifics of the relationship intermediary's information acquisition and signal extraction apparatus, variations in K_i do not affect the intertemporally optimal funding policy as the uninformed outside observer typically might expect it to.

The corresponding independence of perturbations to the project's duration T_i does not hold for all ROC curves and for all decision periods. Firstly, duration independence is limited to non-differentiable two-legged kinked ROC curves of the type proposed in section 2.3.1 for the

relationship intermediary's investment officer. Secondly, whereas some combinations of project duration T_i and ROC curve vertex location $(\alpha_i^v, \varepsilon_i^v)$ do yield duration independence over the project's full lifetime, this will not be the case for all combinations of T_i and $(\alpha_i^v, \varepsilon_i^v)$. Nevertheless it is possible to show that there is a non-degenerate interval $U \in (1, 2, \dots, T_i - 1)$ within which the intertemporally optimal funding policy is constant. Therefore for all decision points that fall within this interval $\tau \in U$ both before and after the duration perturbation, the intertemporally optimal funding policy remains constant and unaffected by the perturbation.

Lemma 3.2 (Duration independence). The optimal funding policy, formalised in the threshold $\theta_{i\tau}^*$ that determines the optimal operating point $(\alpha_{i\tau}^*, \varepsilon_{i\tau}^*)$ for each $\tau \in (1, 2, \dots, T_i - 1)$, is independent of a finite, contemporaneous, permanent perturbation of the project duration T_i . This holds for all ROC curves of the two-legged, piecewise linear, $\text{AUC} > \frac{1}{2}$ kinked form and for all decisions within a non-degenerate interval U of project lifetime $\tau \in U \subset (1, 2, \dots, T_i - 1)$.

Condition 3.1 (Detection of short-termism by uninformed outside observer). An outside observer with neither private information nor access to the relationship intermediary's signal extraction mechanism may rationalise the coincidence of

$$dK_i > 0 \quad \text{and} \quad \Delta \theta_i^* = 0 \quad (3.7a)$$

or

$$dT_i > 0 \quad \text{and} \quad \Delta \theta_i^* = 0 \quad (3.7b)$$

as resulting from excessive discounting of distant cash flows.

In turn, stake independence (lemma 3.1), duration independence (lemma 3.2) and the detection of short-termism by uninformed outside observers (condition 3.1) together imply the observational indistinguishability of (i) short-termism as detected by uninformed outside observers and (ii) the intertemporally optimal, informational rent maximising funding policy. This observational indistinguishability is limited, firstly, to investment officers utilising a two-legged kinked ROC curve of the kind proposed in section 2.3.1, and secondly, to the non-degenerate interval U of project lifetime within which the associated optimal operating point remains constant.

Theorem 3.1 (Indistinguishability). *The relationship intermediary's informational rent maximising, intertemporally optimal funding policy is observationally indistinguishable from short-*

termism within a non-degenerate interval $U \subset (1, 2, \dots, T_i - 1)$ when the investment officer's ROC curve is of the two-legged, piecewise linear, $AUC > \frac{1}{2}$ kinked form.

4 Conclusion

These results indicate that a distortion of the discount rate is not a necessary prerequisite for the emergence of a divergence between different players' optimal investment policies. The usual scapegoat, short-termism, need not be invoked to explain such divergence. Moreover, given that perfect ex ante discrimination ($AUC=1$) between success and failure will not in general be possible, it is natural that a relationship intermediary will on occasion withhold finance when the project would in fact have turned out to be successful in that period. This is merely the consequence of a non-zero false positive likelihood. To the outside observer this again may appear to be indistinguishable from short-termism, but in fact is simply a consequence of imperfect discriminability ($AUC < 1$).

The optimal investment policy for informational rent extraction varies, becoming 'more lenient' in that $P_i^*(\text{nosig})$ is higher, as the share of informational rent deriving from more distant periods increases relative to the share deriving from the proximate period. Technically this variation is a response to changes in the (mis)classification costs C_{TP} , C_{FP} , C_{TN} and C_{FN} , where the extent of the variation is determined by the continuity and shape of the ROC curve. This is not to suggest that the optimal investment policy is independent of the form of the discount function, only that the discount function does not alone drive the optimal investment policy. However, insofar as the relationship intermediary maximises its informational rent, the optimal investment policy is independent of the size of the repeatedly-staked principal K_i .

The detail with which it has been possible to elaborate the relationship intermediary's inner workings here owes much to the adopted Signal Detection Theory (SDT) approach. This approach permits direct examination of how the optimal operating point is to be selected given the trade-off, embodied in the ROC curve, between the true positive likelihood and the false positive likelihood. SDT is particularly suitable here because it exposes and explicitly represents variables and relationships that conventionally remain buried within the deeper structure of Bayesian or NPV calculations. In this sense, adoption of the SDT approach facilitates the refinement to the relationship intermediary's information acquisition and signal extraction apparatus implemented in this paper.

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- binormal-ROCs inv-off-ROC

Figure 1: Binormal ROC curves: firm management and relationship intermediary.

Figure 2: Kinked ROC curve: investment officer's two-state threshold model.

Figure 3: Optimal Operating Points

Aside from the ROC curve forms themselves, these figures assume no more than $\delta_i = .9$ and $p_i = .1$. The ROC curve in (a.) is generated by $f(\theta|\text{succ}) \sim N(0, 1^2)$ and $f(\theta|\text{fail}) \sim N(1, 1^2)$. The vertex of the ROC curve in (b.) is at $(.22, .65)$.

A Technical appendix

A.1 Direct derivation of the optimal operating point

Denoting the direct cost of implementing the generic scoring procedure as $C_{\Gamma(\mathcal{I})}$ and the costs associated with true positives, false negatives, true negatives and false positives as C_{TP} , C_{FN} , C_{TN} and C_{FP} respectively, then the expected cost of using the cutoff threshold θ' as the investment decision criterion takes the form

$$\begin{aligned} E(C_{\theta', \Gamma(\mathcal{I})}) &= C_{\Gamma(\mathcal{I})} + C_{\text{TP}}P_{\theta', \Gamma(\mathcal{I})}(\text{TP}) + C_{\text{FN}}P_{\theta', \Gamma(\mathcal{I})}(\text{FN}) + C_{\text{TN}}P_{\theta', \Gamma(\mathcal{I})}(\text{TN}) + C_{\text{FP}}P_{\theta', \Gamma(\mathcal{I})}(\text{FP}) \\ &= -[C_{\text{FN}} - C_{\text{TP}}]P(\text{fail})P(\text{sig}_{\theta', \Gamma(\mathcal{I})}|\text{fail}) + [C_{\text{FP}} - C_{\text{TN}}]P(\text{succ})P(\text{sig}_{\theta', \Gamma(\mathcal{I})}|\text{succ}) \\ &\quad + C_{\Gamma(\mathcal{I})} + C_{\text{TN}}P(\text{succ}) + C_{\text{FN}}P(\text{fail}) . \end{aligned} \quad (\text{A.1})$$

The optimum threshold θ^* minimises the expected costs of implementing the decision criterion subject to the $\text{TPL}_{\theta', \Gamma(\mathcal{I})}$ and $\text{FPL}_{\theta', \Gamma(\mathcal{I})}$, which are in themselves constrained to lie on the ROC curve generated by $\Gamma(\mathcal{I})$.

$$\theta^* = \arg \min_{\theta'} E(C_{\theta', \Gamma(\mathcal{I})}) \quad (\text{A.2})$$

Hence for ROC curves that are differentiable everywhere, the first order condition

$$\begin{aligned} \frac{dE(C_{\theta', \Gamma(\mathcal{I})})}{d\theta'} &= \\ &= -[C_{\text{FN}} - C_{\text{TP}}]P(\text{fail}) \frac{dP(\text{sig}_{\theta', \Gamma(\mathcal{I})}|\text{fail})}{d\theta'} + [C_{\text{FP}} - C_{\text{TN}}]P(\text{succ}) \frac{dP(\text{sig}_{\theta', \Gamma(\mathcal{I})}|\text{succ})}{d\theta'} = 0 , \end{aligned}$$

gives the slope of the $\text{ROC}_{\Gamma(\mathcal{I})}$ curve at the optimal threshold θ^* as the probability weighted ratio of the cost of success to the cost of failure

$$\frac{dP(\text{sig}_{\theta^*, \Gamma(\mathcal{I})}|\text{fail})}{dP(\text{sig}_{\theta^*, \Gamma(\mathcal{I})}|\text{succ})} = \frac{P(\text{succ}) [C_{\text{FP}} - C_{\text{TN}}]}{P(\text{fail}) [C_{\text{FN}} - C_{\text{TP}}]} . \quad (\text{A.3})$$

This tangency condition simultaneously defines θ^* , $\text{TPL}_{\theta^*, \Gamma(\mathcal{I})}$ and $\text{FPL}_{\theta^*, \Gamma(\mathcal{I})}$. The curvature of the ROC curve therefore determines how much the threshold θ^* and the optimal error ratio $\frac{\beta_{\theta^*, \Gamma(\mathcal{I})}}{\alpha_{\theta^*, \Gamma(\mathcal{I})}}$ change in response to changes in costs. With analogy to production theory, this may be gauged with the elasticity of substitution, here taking the form $\sigma(\Gamma(\mathcal{I})) = d \ln \left(\frac{\beta_{\theta^*, \Gamma(\mathcal{I})}}{\alpha_{\theta^*, \Gamma(\mathcal{I})}} \right) / d \ln \left(\frac{P(\text{succ})}{P(\text{fail})} \frac{[C_{\text{FP}} - C_{\text{TN}}]}{[C_{\text{FN}} - C_{\text{TP}}]} \right)$. Then for all triples $(\Gamma^1, \Gamma^2, \frac{\beta}{\alpha})$ for which $\sigma(\Gamma^2, \frac{\beta}{\alpha}) > \sigma(\Gamma^1, \frac{\beta}{\alpha})$ and $\text{AUC}(\Gamma^2) > \text{AUC}(\Gamma^1)$ it is the less well-informed relationship investor whose optimal response to a given percentage change in probability weighted relative costs will be the greater. This presumes comparability of costs, however. If the driving cost changes in the less

well-informed relationship intermediary are inherently bounded – for instance by the lower absolute level of informational rent it can extract over time due to its lower AUC – then its response in $\Delta \frac{\beta_{\theta^*, \Gamma^1}}{\alpha_{\theta^*, \Gamma^1}}$ will be correspondingly limited in spite of $\sigma(\Gamma^2, \frac{\beta}{\alpha}) > \sigma(\Gamma^1, \frac{\beta}{\alpha})$.

A.2 Informational rent

$$\begin{aligned}
E(\text{NPV} | \varepsilon_i, \alpha_i > 0) - E(\text{NPV} | \varepsilon_i, \alpha_i = 0) &= \mathcal{Q}c(0, 2, K_i, B_i, p_i, \varepsilon_i, \alpha_i) \\
&= -K_i + P_i(\text{succ})B_{i1} + P_i(\text{succ})P_i(\text{nosig}) \left[-K_i + P_i(\text{succ}|\text{nosig})B_{i2} \right] \\
&\quad - \left[-K_i + P_i(\text{succ})B_{i1} + P_i(\text{succ})[-K_i + P_i(\text{succ})B_{i2}] \right] \\
&= -K_i + (1-p_i) \frac{K_i}{(1-p_i)} \\
&\quad + (1-p_i) \left(p_i(1-\varepsilon_i) + (1-p_i)(1-\alpha_i) \right) \left[-K_i + \frac{(1-p_i)(1-\alpha_i)}{p_i(1-\varepsilon_i) + (1-p_i)(1-\alpha_i)} \frac{K_i}{(1-p_i)} \right] \\
&= (1-p_i) \left[- \left(p_i(1-\varepsilon_i) + (1-p_i)(1-\alpha_i) \right) K_i + (1-\alpha_i) K_i \right] \\
&= (1-p_i) p_i (\varepsilon_i - \alpha_i) K_i \quad . \quad (\text{A.4})
\end{aligned}$$

A.3 The optimal operating point

Consider the time T_i-2 case, where $\mathcal{Q}_i(T_i-1, T_i, \varepsilon_{iT_i-1}^*, \alpha_{iT_i-1}^*) = p_i(\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) K_i$. Therefore

$$\begin{aligned}
\left(\frac{d\varepsilon_{iT_i-2}}{d\alpha_{iT_i-2}} \right)_{\bar{C}^*} &= \frac{(1-p_i)}{p_i} \frac{2(\delta_i B_i - K_i + \delta_i p_i (\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) K_i)}{2K_i} \\
&= 1 + \frac{(1-p_i)}{p_i} \frac{\delta_i p_i (\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) K_i}{K_i} = 1 + \delta_i (1-p_i) (\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) \quad . \quad (\text{A.5})
\end{aligned}$$

For the time T_i-3 case, $\mathcal{Q}_i(T_i-2, T_i, \varepsilon_i^*, \alpha_i^*) = p_i(\varepsilon_{iT_i-2}^* - \alpha_{iT_i-2}^*) K_i + p_i(\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) K_i \delta_i (1-p_i) \{ p_i(1-\varepsilon_{iT_i-1}^*) + (1-p_i)(1-\alpha_{iT_i-1}^*) \}$, from which follows that

$$\begin{aligned}
\left(\frac{d\varepsilon_{iT_i-3}}{d\alpha_{iT_i-3}} \right)_{\bar{C}^*} &= 1 + \delta_i (1-p_i) (\varepsilon_{iT_i-2}^* - \alpha_{iT_i-2}^*) \\
&\quad + \delta_i^2 (1-p_i)^2 (\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) \{ p_i(1-\varepsilon_{iT_i-1}^*) + (1-p_i)(1-\alpha_{iT_i-1}^*) \} \quad . \quad (\text{A.6})
\end{aligned}$$

For time T_i-4

$$\begin{aligned}
\left(\frac{d\varepsilon_{iT_i-4}}{d\alpha_{iT_i-4}} \right)_{\bar{C}^*} &= 1 + \delta_i (1-p_i) (\varepsilon_{iT_i-3}^* - \alpha_{iT_i-3}^*) \\
&\quad + \delta_i^2 (1-p_i)^2 (\varepsilon_{iT_i-2}^* - \alpha_{iT_i-2}^*) \{ p_i(1-\varepsilon_{iT_i-2}^*) + (1-p_i)(1-\alpha_{iT_i-2}^*) \} \\
&\quad + \delta_i^3 (1-p_i)^3 (\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) \{ p_i(1-\varepsilon_{iT_i-1}^*) + (1-p_i)(1-\alpha_{iT_i-1}^*) \}^2 \quad . \quad (\text{A.7})
\end{aligned}$$

So for $s \in (1, 2, \dots, T-1)$ arbitrary,

$$\left(\frac{d\varepsilon_{is}}{d\alpha_{is}} \right)_{\bar{C}^*} = 1 + \sum_{t=s+1}^{T-1} \delta_i^{t-s} (1-p_i)^{t-s} (\varepsilon_{it}^* - \alpha_{it}^*) \{p_i(1-\varepsilon_{it}^*) + (1-p_i)(1-\alpha_{it}^*)\}^{t-s-1} . \quad (\text{A.8})$$

A.4 Duration independence

Proof. To show nondegeneracy, it suffices to show that there are at least two proximate periods that share the same optimal operating point. That $T_i - 1 \in U$ holds necessarily, and it remains to show that $T_i - 2 \in U$, i.e. that $\varepsilon_{iT_i-2}^* = \varepsilon_{iT_i-1}^*$ and $\alpha_{iT_i-2}^* = \alpha_{iT_i-1}^*$ for all two-legged ROC curves with the vertex located above the major diagonal $\{(\alpha_i^v, \varepsilon_i^v) : \alpha_i^v, \varepsilon_i^v \in [0, 1] \wedge \varepsilon_i^v > \alpha_i^v\}$. The flatter the lower leg, the smaller the isocost line slope change $\Delta \left(\frac{d\varepsilon_i}{d\alpha_i} \right)_{\bar{C}^*}$ required to induce $\varepsilon_{iT_i-2}^* < \varepsilon_{iT_i-1}^*$ and $\alpha_{iT_i-2}^* < \alpha_{iT_i-1}^*$. The flattest lower leg slope $\frac{\varepsilon_i^v}{\alpha_i^v}$ for any fixed difference $(\varepsilon_i^v - \alpha_i^v)$ occurs where $\varepsilon_i^v \rightarrow 1$. On the other hand, the greatest isocost line slope change $\Delta \left(\frac{d\varepsilon_i}{d\alpha_i} \right)_{\bar{C}^*}$ resulting from any given difference $(\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*)$ is obtained for $\delta_i \rightarrow 1$. Hence $\varepsilon_i^v = 1$ and $\delta_i = 1$ are the most unfavourable conditions under which $\varepsilon_{iT_i-2}^* = \varepsilon_{iT_i-1}^*$ and $\alpha_{iT_i-2}^* = \alpha_{iT_i-1}^*$ may be demonstrated. Once shown for $\varepsilon_i^v = 1$ and $\delta_i = 1$, the result therefore also holds $\forall \varepsilon_i^v, \delta_i \in [0, 1]$. We wish to show that the time $T-2$ isocost line is less steep than the lower leg of the ROC curve

$$1 + \delta_i(1-p_i)(\varepsilon_{iT_i-1}^* - \alpha_{iT_i-1}^*) < \frac{\varepsilon_i^v}{\alpha_i^v} . \quad (\text{A.9})$$

Given the assumed form of the ROC curve, $\varepsilon_{iT_i-1}^* = \varepsilon_i^v$ and $\alpha_{iT_i-1}^* = \alpha_i^v$, and furthermore $\varepsilon_i^v, \delta_i = 1$ for above-mentioned reasons. Therefore (A.9) becomes

$$1 + (1-p_i)(\varepsilon_i^v + 1 - \varepsilon_i^v - (\alpha_i^v + 1 - \varepsilon_i^v)) < \frac{1}{\alpha_i^v + 1 - \varepsilon_i^v} . \quad (\text{A.10})$$

Let $z = 1 - \varepsilon_i^v$, and since $\varepsilon_i^v > \alpha_i^v$ we have $\alpha_i^v + z < 1$. Hence (A.10) simplifies to

$$1 + (1-p_i)(1 - (\alpha_i^v + z)) < \frac{1}{(\alpha_i^v + z)} \quad (\text{A.11})$$

and ultimately to

$$\underbrace{(1-p_i)}_{<1} \underbrace{(\alpha_i^v + z)}_{<1} < 1 , \quad (\text{A.12})$$

from which $\left(\frac{d\varepsilon_{iT_i-2}}{d\alpha_{iT_i-2}} \right)_{\bar{C}^*} < \frac{\varepsilon_i^v}{\alpha_i^v} \forall \varepsilon_i^v, \delta_i \in [0, 1] \Leftrightarrow T_i - 2 \in U$ follows for all two-legged ROC curves with the vertex above the major diagonal $\{(\alpha_i^v, \varepsilon_i^v) : \alpha_i^v, \varepsilon_i^v \in [0, 1] \wedge \varepsilon_i^v > \alpha_i^v\}$. $\square$

A.5 Indistinguishability

Proof. Stake independence, by Lemma 3.1, fulfils detection condition (3.7a). Duration independence, by Lemma 3.2, fulfils detection condition (3.7b). $\square$